



## Earth-ionosphere wave guide model for Rugby – Visakhapatnam path consistent with VLF phase variation measurements

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**Abstract** : The phase variation measurements of 16 kHz VLF transmissions from Rugby (England) made at Visakhapatnam (India) for over one year have been used as the basis to find the model for the Earth – ionosphere wave guide for the transmission path that reconciles with experimental measurements. The computations have been carried out assuming a perfectly conducting Earth and three different lower ionospheric conductivity models, namely, sharply bounded with infinite conductivity, sharply bounded with finite conductivity and a diffuse boundary with exponentially varying conductivity. It is found that the exponential model gives a diurnal phase shift or day to night shift in the height of reflection much less than what is obtained from measurements. But a sharply bounded ionosphere with finite conductivity is found to reproduce not only the diurnal height change but also the seasonal variation in it, seen in experimental data.

**Keywords** Radio wave propagation, ionosphere, D-region, VLF phase variations

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It is not very long ago that almost all long distance communications were by means of sky wave. However, in recent times, there has been an increase in use of satellites for communication and navigation. These systems operate at frequencies on which the ionospheric effects are minimal and hence, the interest in ionosphere as a communication channel (especially for HF) has lessened. This, however, does not mean that ionospheric propagation is no longer important since other services like broadcasting, ship and aircraft communication *etc.* still rely on ionospheric propagation. Further, the vulnerability of the hardware of satellite systems to hazards in space and degrading influence of vagaries in space weather in satellite signals emphasize that ground-based systems are reliable alternatives and bring back the interest to ionosphere. The D-region of the ionosphere lying at its base has profound influence on the character of radio waves passing through it particularly in the very low, low, medium, and high frequency (VLF, LF, MF and HF) bands which are still the principal bands used in communication and broadcasting. Because of the complexities

in its chemistry and dynamical behavior, the D-region continues to be an area of intense investigation. In this context, it may be noted that one of the goals of recent TIMED (Thermosphere, Ionosphere Mesosphere Energetics satellite) Project of NASA is D-region investigations [1]. In the study of the D-region, ground-based propagation experiments are still considered invaluable, since they provide reliable information on long term basis. Among the many ground-based techniques, the recording of phase and amplitude of long distance VLF (3-30 kHz) signals still proves to be best suited, since the reflection point for these waves lies entirely in the D-region both during day and night and uncertainties involved in separating D- and E-region contributions inherent in medium and high frequency observations, do not arise.

Many aspects of VLF wave propagation to great distances are generally explained in terms of the wave guide theory [2]. The lower ionosphere, namely, the D-region and the surface of the Earth, assumed to be perfectly conducting, act as the walls of the wave guide through which the signal travels from the transmitter to the receiver. Although a number of modes are

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excited when a VLF wave propagates in the Earth-ionosphere wave guide, at long distances, only the first order mode remains significant since higher order modes get attenuated [3]. In this Earth – ionosphere wave guide, the lower ionosphere (D-region) acting as a reflecting layer of VLF waves is described by different models, viz., sharply bounded with finite conductivity, sharply bounded with infinite conductivity or a diffuse boundary with exponentially varying conductivity. Different workers considered one or more of these models of the ionosphere to explain their experimental measurements of diurnal phase / amplitude variations. For example, Crombie [4] and Scarabucci and Mendonca [5] employed a homogeneous model while Wait and Spies [6], Kikuchi [7] and Joshi and Iyer [8] considered exponentially-varying ionosphere to explain their experimental results. It is the purpose of the present work to reconcile the experimental measurements of diurnal phase shift or shift in reflection height of 16 kHz Rugby transmissions received at Visakhapatnam with computed values using the models described above. The model that gives the values consistent with the experimentally measured ones is considered appropriate for the propagation path.

The phase variation measurements of 16 kHz transmissions from Rugby (52.3°N, 1.2°W), England made at Visakhapatnam (17.7°N, 83.3°E) during November, 1984 – October, 1985 have been considered in this study. The locations of transmitting and receiving stations and the propagation path are shown in Figure 1.

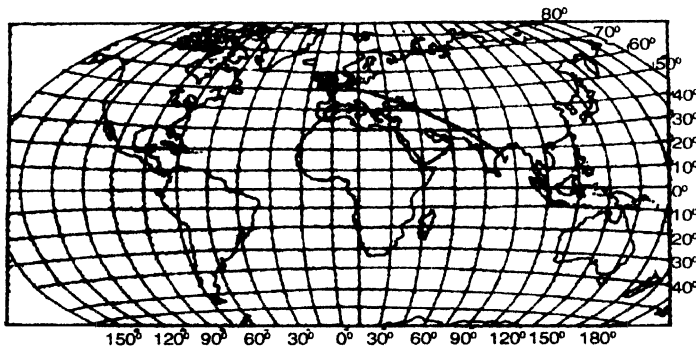


Figure 1. Propagation path between Rugby (R) and Visakhapatnam (V).

The technique of measurement is coherent phase detection technique in which the incoming 16 kHz signal is received by a Tracor 900A VLF receiver and the signal phase is compared with that of a signal obtained from a highly stable standard frequency generator type HP(5065A) Rubidium Vapor Frequency Standard by which an accuracy of 1 part in  $10^{11}$  is normally exhibited by the receiver.

In Figure 2, the monthly mean diurnal phase variations are shown. The sunrise and sunset times at the transmitter and receiver are indicated by arrows. The diurnal phase curves exhibit typical characteristics with the propagation time fairly

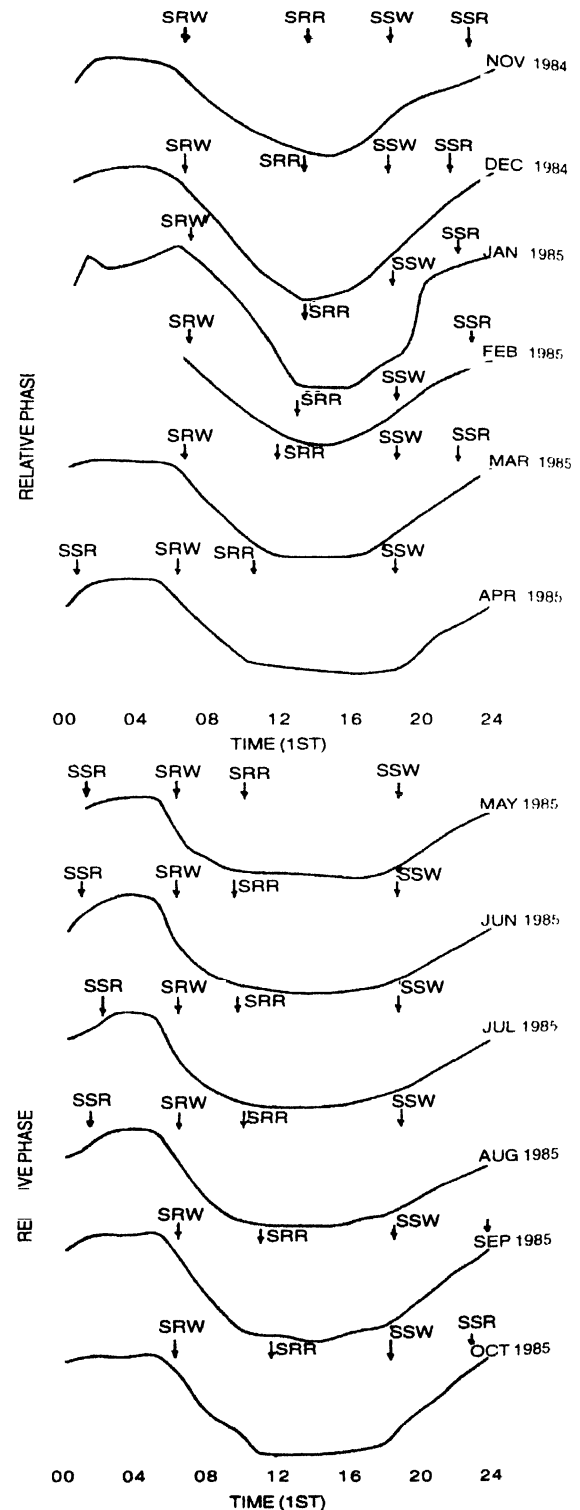


Figure 2. Monthly mean diurnal phase variations (SRW, SSW, SRR. SSR refer to sunrise and sunset times at Visakhapatnam and Rugby respectively)

steady and greater when both the transmitter and receiver are in darkness, than when both are in day light. This typical trapezoidal phase variation implies that the first order mode is predominant. These monthly mean phase change data are used to derive one of the important quantities, namely, the transmission delay or diurnal change in transmission time, which is the phase difference of the signal between day and night. This diurnal change in transmission time is found to vary from 7.2  $\mu\text{s/Mm}$  in April to 11.7  $\mu\text{s/Mm}$  in December with an annual mean of 8.5  $\mu\text{s/Mm}$  which value compares well with those obtained by Kikuchi [9] and Joshi and Iyer [8] for VLF signals propagating over similar path lengths.

This diurnal change in the phase of VLF waves is due to change in phase velocity brought about by a change in the height of reflection between night and day. Hence, the diurnal shift in height of reflection;  $\Delta h$ , can be calculated from the diurnal phase shift using the relation [3]

$$\Delta h = -\frac{c\Delta t}{d} \left[ \frac{\lambda^2}{16h_0^3} + \frac{1}{2a} \right]^{-1} \text{ km}, \quad (1)$$

where  $c$  is the velocity of light,  $h_0$  - the height of the day time reflecting layer,  $a$  - the radius of the earth,  $d$  - the distance of the receiver from the transmitter,  $\lambda$  - the wavelength of the VLF signal and  $\Delta t$  - the phase change in seconds (all distances / lengths given in kilometers). Taking the daytime height of reflection to be 70 km, the day to night change in reflection height has been computed for each month. The monthly mean values are grouped season wise and the average value in each season is presented in Table 1.

Table 1. Seasonal variation day-to-night shift in reflection height ( $\Delta h$ ) from VLF phase measurements.

Season	$\Delta h$ (km)
Winter	20.5
Equinox	17.6
Summer	15.7
Annual	18.0

It is evident that the diurnal shift in reflection height shows seasonal variation with maximum in winter. A similar seasonal trend in  $\Delta h$  has been reported by Olga *et al* [10] in the phase variation data of 12.9 kHz signal. As the daytime D-region is fairly stable, the rise in day to night shift in reflection height in winter suggests a variation in the night time electron density brought about by changes in minor neutral constituents [11].

In the waveguide mode theory for VLF waves, if  $v_{1n}$  and  $v_{1d}$  are the phase velocities of the VLF waves for the all night and all day paths respectively, the diurnal phase shift ( $\Phi_{dn}$ ) is given by [2]

$$\Phi_{dn} = \frac{1}{3} \left( \frac{c}{v_{1n}} - \frac{c}{v_{1d}} \right) 10^4 \mu\text{s/Mm}, \quad (2)$$

where  $c$  is the velocity of light. The first order mode phase velocity  $v_1$  is given by

$$\frac{v_1}{c} = 1 + \text{Re } C_1^2 \quad (3)$$

$C_1^2$  is given by

$$C_1^2 = \frac{7\pi}{6} \cdot \frac{2ka}{3} \left[ \left( \frac{2h}{a} \right)^{\frac{3}{2}} - i\alpha \left( \frac{2h}{a} \right)^{\frac{1}{2}} \right] + i \left[ \frac{\alpha}{2} \right] \left[ \left( \frac{a}{2h} \right)^{\frac{1}{2}} \right] \quad (4)$$

in which  $h$  is the height of reflection of the VLF wave in the ionosphere,  $a$  is the radius of the earth,  $\lambda$  is the wavelength,  $k = 2\pi/\lambda$  and  $\alpha = -2i^{1/2}(\omega/\omega_r)^{1/2}(1-i\omega_r/\omega)$  where  $\omega$  is the angular frequency of the VLF wave and  $\omega_r$ , the ionospheric conductivity parameter given by  $\omega_r = \omega_p^2/\nu$ ,  $\omega_p$  and  $\nu$  being the angular frequency and the effective collision frequency, respectively.

Using the wave guide mode equations for a perfectly conducting ground and infinitely conducting or sharply bounded ionosphere with a finite conductivity, it is possible to obtain the diurnal phase shift theoretically and thereby estimate the night time reflection levels assuming its day time value. For a given ionospheric model, the conductivity parameter is specified and using the above theory, phase velocities and the diurnal phase shift can be computed.

In the present investigation, the phase variation is calculated using a wave guide model consisting of a perfectly conducting Earth and an ionosphere which is (i) infinitely conducting, (ii) sharply bounded with finite conductivity ( $\omega_r = 2 \times 10^5$ ) and (iii) with exponentially varying conductivity. In the case of exponential model, the conductivity parameter assumed is identical to that suggested by Wait [12] viz.,  $\omega_r = 2.5 \times 10^5 \exp[\beta(z-h)]$ , where  $\beta$  is the height gradient and  $h$  is the reference height.

Assuming the height gradient to be 0.3  $\text{km}^{-1}$  for day time and 0.5  $\text{km}^{-1}$  for night time and taking the day time reference height ( $h_d$ ) to be 70 km, the diurnal phase shift has been calculated as a function of  $h_n$ , the night time reference height. The night time reference height where the computed  $\Phi_{dn}$  agrees with the measured value for each of the three ionospheric models, gives

the night time reflection height ( $h_n$ ) and the difference ( $h_n - h_d$ ) gives the diurnal shift in reflection height ( $\Delta h$ ). The results of the computation of  $\Delta h$  for the three models in three seasons are shown in Table 2.

**Table 2.** Computed values of  $\Delta h$  (km) in different ionospheric models.

Season	Sharply bounded + Infinite conductivity	Sharply bounded+ finite conductivity	Diffuse layer with Exponential conductivity
Winter	23.4	20.4	10.4
Equinox	18.9	16.3	9.65
Summer	19.9	17.5	9.75
Annual	21.0	18.0	10.0

It may be pointed out here that the  $\Delta h$  ( $= 10$  km) obtained in the case of an exponentially varying ionospheric conductivity, agrees very well with 11.5 km obtained by Joshi and Iyer [8]. However, the exponential model is not able to reproduce the seasonal variation in  $\Delta h$  seen in the experimental data. But the model with a sharply bounded ionosphere with finite conductivity not only gives  $\Delta h$  value in agreement with that obtained from measured phase deviations but also reproduces seasonal variation in it.

Wave guide model analysis for Earth – ionosphere wave guide for VLF propagation has been carried out to compute the diurnal phase shift of 16 kHz signals assuming perfectly conducting Earth and three different lower ionospheric

conductivity models. It is found that a wave guide model with a perfectly conducting Earth and sharply bounded ionosphere with finite conductivity as the two walls of the guide, gives results consistent with the experimentally measured diurnal phase variation of 16 kHz Rugby (England) transmissions recorded at Visakhapatnam (India).

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